Disorder and temperature renormalization of interaction contribution to the conductivity in two-dimensional $In_xGa_{1-x}As$ electron systems

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We study the electron-electron interaction contribution to the conductivity of two-dimensional $In_{0.2}Ga_{0.8}As$ electron systems in the diffusion regime over the wide conductivity range, $\sigma \simeq (1-150)G_0$, where $G_0 = e^2/\pi h$. We show that the data are well described within the framework of the one-loop approximation of the renormalization-group (RG) theory when the conductivity is relatively high, $\sigma \gtrsim 15G_0$. At lower conductivity, the experimental results are found to be in drastic disagreement with the predictions of this theory. The theory predicts much stronger renormalization of the Landau's Fermi-liquid amplitude, which controls the interaction in the triplet channel, than that observed experimentally. A further contradiction is that the experimental value of the interaction contribution does not practically depend on the magnetic field, whereas the RG theory forecasts its strong decrease due to decreasing diagonal component of the conductivity tensor in the growing magnetic field.

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I. INTRODUCTION

A contribution of electron-electron (e-e) interaction to the conductivity is studied since 1980.[1,2] At high value of the Drude conductivity, $\sigma_0 = \pi k_F l G_0 \gg G_0$, where k_F is Fermi quasimomentum, l is the mean-free path, and $G_0 = e^2/\pi h$, and in the diffusion regime, $T\tau \ll 1$, where τ is transport relaxation time, this contribution is,

$$\delta\sigma_{ee} = K_{ee} G_0 \ln(T\tau),$$

$$K_{ee} = 1 + 3\left[1 - \frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2)\right], \quad (1)$$

where γ_2 stands for the Landau's Fermi-liquid amplitude. The coefficient K_{ee} has two terms coming from singlet and triplet channels [the first and second terms in Eq. (1), respectively]. They are opposite in sign favoring localization and antilocalization, respectively. In conventional conductors the γ_2 value is small and the net effect is in favor of localization. Together with the weak localization (WL) it leads to dielectric behavior of the conductivity, σ, $d\sigma/dT > 0$. However, the analysis of the e-e interaction contribution performed in the framework of the theory of the renormalization group (RG) (Refs. 3–8) shows that the reduction in the temperature and/or conductivity should lead to renormalization of the Fermi-liquid amplitude γ_2. At $\sigma_0 \lesssim (5-15)G_0$ or in dilute systems this amplitude may be significantly enhanced due to e-e correlations that can result in a metalliclike T dependence of the conductivity, $d\sigma/dT < 0$. The theoretical study within the one-loop approximation for arbitrary valley degeneracy n_v was carried out in Refs. 7 and 8. The role of two-loop diagrams was studied for two cases only. The first case relates to multivalley systems ($n_v \gg 1$) with $\gamma_2 \ll 1$.[8] The second one is single-valley system ($n_v = 1$) with large γ_2 value.[9] The RG theory has been used with advantage for understanding of the temperature dependence of the conductivity and metal-insulator transition in Si-metal-oxide-semiconductor field-effect transistors.[7,10–12] As far as we know there are no experimental study to examine the region of validity of this theory for the simplest two-dimensional (2D) systems with the single-valley isotropic spectrum in the deeply diffusion regime for which the RG equations were derived.

Besides, the analysis of the temperature dependence of the conductivity at $B=0$ alone is not the reliable way to understand the role of the renormalization of e-e interaction and range of validity of the one-loop approximation. It is because there are lots of effects, such as the weak localization and antilocalization, the ballistic contribution of the e-e interaction, the temperature-dependent screening, the temperature-dependent disorder, and so on, which govern the temperature dependence of the conductivity along with the e-e interaction. Certain of these effects are poorly controlled. Experimentally, it manifests itself as that the values of the interaction contribution to the conductivity found from the temperature dependence of conductivity at $B=0$ and at $B \neq 0$ are significantly different even in the case of high conductivity.[13]

From our point of view the reliable results can be obtained only from simultaneous analysis of the data obtained at $B=0$ and at low and high magnetic fields. The unique property of the e-e interaction in the diffusion regime is the fact that it contributes to the diagonal component of the conductivity tensor, σ_{xx}, only. Just this feature gives a possibility to obtain experimentally the e-e interaction contribution to the conductivity even for the low conductivity when the interference contribution dominates.[14,15] Following this line of attack and analyzing the experimental results obtained for the 2D electron gas in the $GaAs/In_{0.2}Ga_{0.8}As/GaAs$ single-quantum well, the authors of Ref. 14 come to the conclusion that the temperature dependence of the e-e interaction contribution remains logarithmical over the wide conductivity range, $\sigma_0 \simeq (1\ldots100)G_0$: $\delta\sigma_{ee} \simeq K_{ee} G_0 \ln(T\tau)$. However, the coefficient K_{ee} is found dependent on the disorder strength. Its value drastically decreases when σ_0 decreases, starting from $\sigma_0 \simeq (12-15)G_0$. Although this effect is prominent, it was not since discussed and its origin remains unclear.

In this paper we report the results of the detailed study of the conductivity of 2D electron gas in $In_{0.2}Ga_{0.8}As$ and GaAs single-quantum well at $B=0$ and $B\neq 0$ over the wide conductivity range. We begin by considering the predictions of the RG theory. Then, after description of experimental details, we will outline the procedure used for extracting the diffusion part of the interaction correction. Finally, analyzing the temperature dependences of the interaction contribution and the conductivity we will show that the one-loop approximation adequately describes the data while $\sigma \gtrsim 15G_0$ and strongly disagrees with that at lower conductivity. The conflict between the experiment and RG theory arising in the presence of the magnetic field will be discussed as well.

II. PREDICTIONS OF THE RG THEORY

Before considering and discussing the experimental results let us demonstrate the role of the γ_2 renormalization.

The temperature dependence of σ and γ_2 is described in the framework of one-loop approximation of RG theory by the following system of the differential equations,[3–8]

$$\frac{d\sigma}{d\xi} = -\left\{1+1+3\left[1-\frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2)\right]\right\}, \qquad (2)$$

$$\frac{d\gamma_2}{d\xi} = \frac{1}{\sigma}\frac{(1+\gamma_2)^2}{2}, \qquad (3)$$

where $\xi=-\ln(T\tau)$ and σ is measured in units of G_0. The quantity γ_2 is expressed through the Fermi-liquid constant F_0^σ, $\gamma_2=-F_0^\sigma/(1+F_0^\sigma)$. For the high conductivity, the value of F_0^σ depends on the gas parameters $r_s=\sqrt{2}/(a_B k_F)$, where a_B is the effective Bohr radius, and for small r_s values is[16]

$$F_0^\sigma = -\frac{1}{2\pi}\frac{r_s}{\sqrt{2-r_s^2}}\ln\left(\frac{\sqrt{2}+\sqrt{2-r_s^2}}{\sqrt{2}-\sqrt{2-r_s^2}}\right), \quad r_s^2<2. \qquad (4)$$

The term $1+1$ in braces in Eq. (2) is responsible for the weak localization and the interaction in singlet channel, which in the case of Coulomb interaction give equal contributions. Equation (3) describes the renormalization of the Landau's Fermi-liquid amplitude γ_2 with the temperature and conductivity. One can see from Eq. (3) that the γ_2 renormalization can be neglected at high conductivity. In this case the integration of Eq. (2) gives

$$\sigma(T) = (1+K_{ee})G_0 \ln(T\tau) + \text{const} \qquad (5)$$

with K_{ee} given by Eq. (1). This expression accords well with the known expression

$$\sigma(T) = \sigma_0 + K_{ee}G_0 \ln(T\tau) + G_0 \ln\left[\frac{\tau}{\tau_\phi(T)}\right], \qquad (6)$$

where $\tau_\phi(T) \propto 1/T$ is the phase relaxation time controlled in 2D systems by the inelasticity of the *e-e* interaction. However, Eqs. (2) and (3) predict that the change in the amplitude γ_2 and, hence, the deviation of the temperature dependence of the conductivity from the logarithmic one is appreciable already at moderate conductivity value. The written is illustrated by Fig. 1, in which the results of numerical solution of Eqs. (2) and (3) are presented. We used the parameters, which are typical for the moderately disordered $GaAs/In_{0.2}Ga_{0.8}As/GaAs$ heterostructures investigated in this paper. The minimal $T\tau$ value corresponds to $T=0.1$ K for all the cases. The following initial conditions have been used. We suppose that the high-temperature conductivity is equal to the Drude conductivity, $\sigma(\xi=0)=\sigma_0$. This condition seems to be natural. It corresponds to that the diffusion part of interaction correction is equal to zero and the WL correction is much less than the Drude conductivity at $T\tau=1$. The second condition is $\gamma_2(\xi=0)\equiv\gamma_2^0=-F_0^\sigma/(1+F_0^\sigma)$, where F_0^σ is determined by Eq. (4).

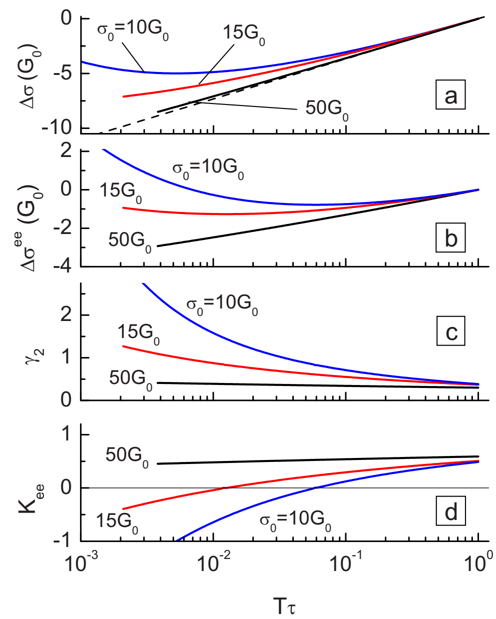

FIG. 1. (Color online) (a) The temperature dependences of the conductivity change $\Delta\sigma=\sigma(T\tau)-\sigma(T\tau=1)$, (b) the *e-e* interaction contribution to the conductivity, (c) the Fermi-liquid amplitude γ_2, and (d) K_{ee} found from the solution of Eqs. (2) and (3). The dashed line in panel (a) is the dependence $1.59G_0 \ln(T\tau)$. The following parameters have been used, $n=2.0\times10^{11}$, 2.5×10^{11}, and 5.0×10^{11} cm^{-2} and γ_2^0=0.383, 0.367, and 0.3 for $\sigma_0=10G_0$, $15G_0$, and $50G_0$, respectively.

One can see from Fig. 1(c) that the renormalization of γ_2 for the high Drude conductivity, $\sigma_0=50G_0$, is rather small so that the temperature dependence of the conductivity is close to the logarithmic one with the slope determined by the initial value of γ_2, $\Delta\sigma(T)=[1+K_{ee}(\gamma_2^0)]G_0 \ln T\tau=1.59G_0 \ln T\tau$ [Fig. 1(a)]. Nevertheless, the noticeable decrease in the K_{ee} value with the lowering temperature is evident even for so high conductivity [see Fig. 1(d)]. The K_{ee} value at $T\tau=3.5\times10^{-3}$ is approximately equal to 0.46 while K_{ee} at $T\tau=1$ is close to 0.6.

For the lower Drude conductivity, $\sigma_0=15G_0$, the renormalization of γ_2 with the temperature decrease becomes significant [Fig. 1(c)]. The sign of $d\Delta\sigma^{ee}/dT$ is changed at $T\tau\simeq0.012$ from positive at high temperature to negative at lower one [Fig. 1(b)]. However, the temperature dependence of the overall conductivity remains insulating ($d\sigma/dT>0$)

due to dominating WL contribution. Finally, for $\sigma_0=10G_0$, the renormalization of γ_2 is so huge [Fig. 1(c)] that the metallic behavior of the interaction correction [Fig. 1(b)] wins the insulating behavior of the WL correction at low temperature and, as consequence, the total conductivity behaves itself metallically at $T\tau \lesssim 5\times 10^{-3}$ [Fig. 1(a)]. To the best of our knowledge such the behavior was never experimentally observed in the moderately disordered 2D systems of weakly interacting electrons with the simple single-valley energy spectrum, characterizing by $r_s<2-3$ and $\sigma \gtrsim 1G_0$. The goal of this paper is to examine how the one-loop approximation describes the experimental data for such the systems and, thus, establish the region of validity of this theory.

III. EXPERIMENT

The results of experimental study of the evolution of the diffusion part of the interaction correction to the conductivity in a n-type 2D system with decreasing Drude conductivity within the range from $\sigma_0 \simeq 150G_0$ to $\sigma_0 \simeq 5G_0$ at the temperatures when $T\tau < 0.1-0.15$ are reported. The ballistic contribution of the e-e interaction is small under these conditions. The data for two structures, 3510 and 4261, are analyzed. The structure 3510 with moderate disorder has two δ-doping layers disposed in the barriers on each side of the quantum well on the distance of about 9 nm. The structure 4261 with higher disorder has the δ layer in the center of the quantum well. In more detail the structures design is described in Refs. 17 and 18. The electron density n and mobility μ in the structures are as follows: $n=7.0\times 10^{11}$ cm^{-2}, $\mu=19300$ cm^2/Vs for structure 3510 and $n=1.8\times 10^{12}$ cm^{-2}, $\mu=1600$ cm^2/Vs for structure 4261. The samples were mesa etched into standard Hall bars and then an Al gate was deposited by thermal evaporation onto the cap through a mask. Varying the gate voltage, we changed the electron density in the quantum well and changed the conductivity from its maximal value down to $\sigma \simeq 1G_0$.

First, let us demonstrate that the structures investigated are "normal," i.e., the transport in zero, low and high magnetic field at the high conductivity, when the renormalization of the e-e interaction should be negligible, is consistent with the following simple model. The temperature dependence of the conductivity in the absence of magnetic field can be described by Eq. (6), whereas in the presence of the magnetic field the conductivity tensor components are

$$\sigma_{xx}(B,T) = \frac{en\mu(B,T)}{1+[\mu(B,T)B]^2} + \delta\sigma^{ee}(T), \qquad (7)$$

$$\sigma_{xy}(B,T) = \frac{en\mu(B,T)^2 B}{1+[\mu(B,T)B]^2}. \qquad (8)$$

Because the WL correction is actually reduced to the renormalization of the transport relaxation time,[19] it is incorporated here into the mobility in such a way that

$$\delta\sigma^{\mathrm{WL}}(B,T) = en\,\delta\mu(B,T), \qquad (9)$$

where

$$\delta\sigma^{\mathrm{WL}}(B=0,T) = -G_0 \ln\left[\frac{\tau}{\tau_\phi(T)}\right], \qquad (10)$$

and $\Delta\sigma^{\mathrm{WL}}(B)=\delta\sigma^{\mathrm{WL}}(B)-\delta\sigma^{\mathrm{WL}}(B=0)$ is described by the expression[20,21]

$$\Delta\sigma^{\mathrm{WL}}(B) = \alpha G_0 \mathcal{H}\left(\frac{\tau}{\tau_\phi}, \frac{B}{B_{tr}}\right),$$

$$\mathcal{H}(x,y) = \psi\left(\frac{1}{2}+\frac{x}{y}\right) - \psi\left(\frac{1}{2}+\frac{1}{y}\right) - \ln x. \qquad (11)$$

Here, $B_{tr}=\hbar/(2el^2)$ is the transport magnetic field, $\psi(x)$ is a digamma function, and α is the prefactor, whose value depends on the conductivity if one takes into account two-loop-localization correction and the interplay of the weak localization and interaction, $\alpha=1-2G_0/\sigma$.[22,23]

For structure 3510, the low-field magnetoconductivity $1/\rho_{xx}(B)$, which results from the suppression of the interference quantum correction, measured at high conductivity for different temperatures is presented in inset in Fig. 2(a). One can see that the data are well described by Eq. (11). The temperature dependence of τ_ϕ within the experimental accuracy is close to $1/T$ [Fig. 2(a)]. This shows that the main mechanism of the phase relaxation is, as expected, inelasticity of the e-e scattering. The prefactor value is close to unity.

To find the diffusion part of the interaction correction we take approach which has been detailed in our previous paper, Ref. 17. It uses the unique property of the diffusion correction to contribute to σ_{xx} and not to σ_{xy} [see Eqs. (7) and (8)]. Thus, in order to obtain the correction experimentally, one should find such a contribution to the conductivity which exists in σ_{xx} but is absent in σ_{xy}. The temperature dependences of $\Delta\sigma_{xx}^{ee}=\delta\sigma_{xx}^{ee}(T)-\delta\sigma_{xx}^{ee}(1.35\ \mathrm{K})$ found in such a way

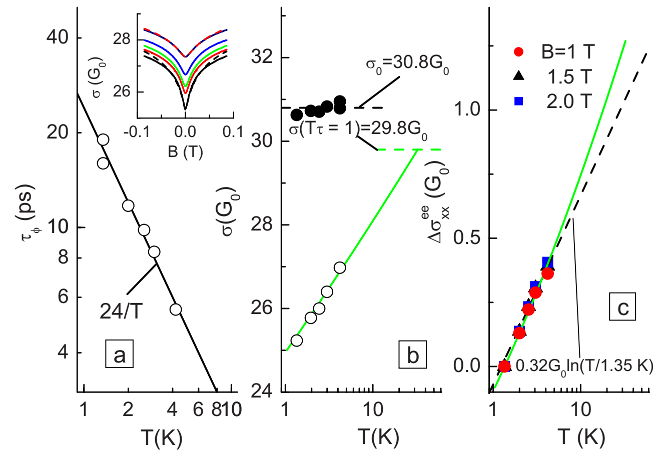

FIG. 2. (Color online) (a) The temperature dependences of the phase relaxation time, (b) σ and σ_0 found from experiment (see text), and (c) $\Delta\sigma_{xx}^{ee}$ taken at different magnetic field. Inset in panel (a) shows the σ vs B dependence for T=4.2, 3.0, 2.56, 2.0, 1.35 K (from top to bottom). The dashed lines are the results of the best fit by Eq. (11) carried out at $|B|<0.3B_{tr}$ (B_{tr}=80 mT for this case). Solid lines in (b) and (c) are the solutions of the RG equations with the initial conditions, $\sigma(T\tau=1)=29.8G_0$, $\gamma_2^0=0.4$. Structure 3510, $n=3.35\times 10^{11}$ cm^{-2}.

for the different magnetic fields are shown in Fig. 2(c). One can see that these dependences are logarithmic within the experimental accuracy, $\Delta\sigma_{xx}^{ee}=K_{ee}G_0 \ln(T/1.35\ \text{K})$ and $K_{ee} \simeq 0.32$ does not depend on the magnetic field. The temperature dependence of the conductivity at $B=0$ is shown in Fig. 2(b). As seen it is also logarithmic, and, what is more important, the slope of the σ vs $\ln T$ dependence is close to the value $1+K_{ee}=1.32$ predicted theoretically [see Eq. (5)]. This fact justifies that there are no additional mechanisms of the T dependence of the conductivity in the samples investigated. It is wholly determined by the temperature dependence of the WL and interaction quantum corrections. Now, knowing the experimental K_{ee} and τ_ϕ values one can easily estimate the value of the Drude conductivity with the use of Eq. (6). As seen from Fig. 2(b) the values of σ_0 found at different temperatures are very close to each other. This attests that the model is adequate and the value of σ_0 found in this way is a good estimate for the Drude conductivity. Thus, $\sigma_0 =(30.8\pm 0.2)G_0$ for this case.

Let us compare the experimental temperature dependences of conductivity with that predicted by the RG theory. Solid line in Fig. 2(b) is the result of the numerical solution of Eqs. (2) and (3) with the initial parameters which give the best fit of the data, $\sigma(T\tau=1)=29.8G_0$ and $\gamma_2^0=0.4$. The variation in the e-e interaction contribution shown in Fig. 2(c) has been obtained by subtraction of the WL contribution from the calculated σ vs T curve as follows: $\Delta\sigma^{ee}(T)=\sigma(T) -\sigma(1.35\ \text{K})-\ln(T/1.35\ \text{K})$. Excellent agreement between the data and solution of the RG equations is evident both for $\sigma(T)$ and $\Delta\sigma_{xx}^{ee}(T)$.

It is worth noting that the values of $\sigma(T\tau=1)$ and γ_2^0 found from the fit are reasonable. The initial value of γ_2, $\gamma_2^0=0.42$, is close to that calculated from Eq. (4), $\gamma_2=0.37$. The value of σ at $T\tau=1$ is less than the Drude conductivity estimated experimentally by the value of about $1G_0$. The reason is that not all the interference quantum correction is suppressed at $T\tau=1$. Really, if one extrapolates the experimental τ_ϕ vs T dependence to $T=\tau^{-1}\simeq 30\ \text{K}$ we obtain for the rest of $\delta\sigma^{\text{WL}}$ at $T\tau=1$: $\delta\sigma^{\text{WL}}(30\ \text{K})\simeq G_0 \ln[\tau/\tau_\phi(30\ \text{K})]\simeq -1.2G_0$. Thus, the Drude conductivity estimated as $\sigma(30\ \text{K}) -\delta\sigma^{\text{WL}}(30\ \text{K})$ is $\sigma_0\simeq(29.8+1.2)G_0=31G_0$. This value practically coincides with that obtained above, $\sigma_0 =(30.8\pm 0.2)G_0$.

It would be fine to trace experimentally the γ_2 change over the whole temperature range starting from $T\tau=1$. However, the ballistic contribution of the interaction correction, the partial lifting of the degeneracy of the electron gas, finally, the phonon scattering controls the temperature dependence of the conductivity at high temperature. All this makes it impossible to determine the e-e interaction contribution accurately already at $T\tau\gtrsim 0.1-0.15$. On the other hand, Fig. 1 shows that the renormalization of γ_2 strongly depends on the value of the Drude conductivity, it should be more pronounced at lower Drude conductivity. Let us, therefore, consider what happens with the lowering of the conductivity.

The analysis described above has been carried out over the wide range of the gate voltage which controls the electron density, the mobility and, thus, the Drude conductivity. All the dependences, namely, $\sigma(T)$ at $B=0$, $\tau_\phi(T)$, and $\sigma_{xx}(T)$ are similar to that shown in Fig. 2 down to $\sigma\simeq 1G_0$. However, agreement of the data with the solution of Eqs. (2) and (3) is worse, the lower the conductivity.

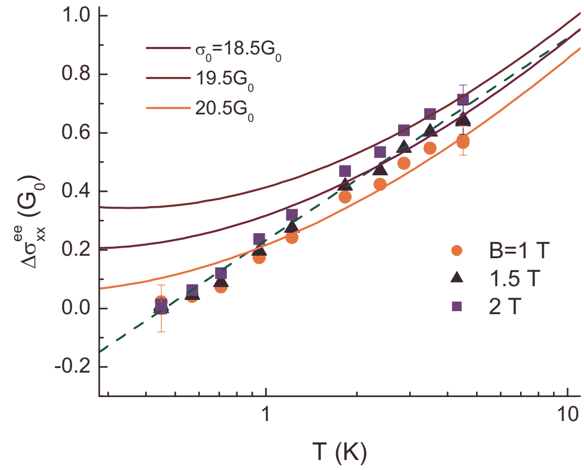

FIG. 3. (Color online) The temperature dependences of $\Delta\sigma_{xx}^{ee}$ for different magnetic fields for structure 3510 at $\sigma_0=18.5G_0$ and $n =3\times 10^{11}\ \text{cm}^{-2}$. The curves are the solutions of the RG equations for different σ_0 and $\gamma_2^0=0.41$. For clarity the curves are shifted in vertical direction. The dashed line is the dependence $0.3G_0 \ln(T/0.45\ \text{K})$.

Disagreement becomes noticeable already at $\sigma_0\simeq 19G_0$. It is more visible in $\Delta\sigma_{xx}^{ee}$ vs T dependence (see Fig. 3). The experimental dependence remains close to the logarithmic one while the curve calculated with the initial value $\sigma(T\tau =1)=\sigma_0+\delta\sigma^{\text{WL}}(T\tau=1)=18.5G_0$ shows upturn at $T\lesssim 0.3\ \text{K}$. The γ_2 value found from the slope of the experimental dependence is approximately equal to 0.55, whereas the calculated value of γ_2 changes from $\gamma_2=0.57$ at $T=4.2\ \text{K}$ to $\gamma_2 =0.8$ at $T=0.45\ \text{K}$. Variation in the initial conditions within the reasonable range does not improve agreement. The dependence calculated remains nonlogarithmic.

The distinction between the calculation and experimental results becomes more clear at lower σ_0. As an example we present the data for $\sigma_0\simeq 9.6G_0$ in Fig. 4. It is seen from Fig. 4(b) that the temperature dependence of the conductivity remains close to the logarithmic one. The temperature dependence of $\Delta\sigma_{xx}^{ee}$ is also close to the logarithmic one with the slope corresponding to $\gamma_2=0.64-0.68$.

As in the case of the higher conductivity, the temperature dependence of τ_ϕ is close to $1/T$ [Fig. 4(a)] therefore the term responsible for the weak localization in Eq. (2) remains equal to 1. However, it is impossible to describe the temperature dependence of the conductivity for $B=0$ if one uses $\sigma(T\tau=1)$ found from the experimental Drude conductivity as in the previous case [Fig. 4(b)]. One can suppose that the value of $\sigma(T\tau=1)$ has been obtained with large error and another value should be used as initial one. We tried to describe the data using both $\sigma(T\tau=1)$ and γ_2^0 as the fitting parameters. As seen from Fig. 4(b) the much better agreement can be achieved in this case. However, as clearly evident from Fig. 4(c), even with these parameters the calculated dependence $\Delta\sigma_{xx}^{ee}(T)$ strongly deviates from the experimental one. Namely, the upturn of $\Delta\sigma_{xx}^{ee}$ with the temperature decrease predicted by the RG theory is not observed experimentally.

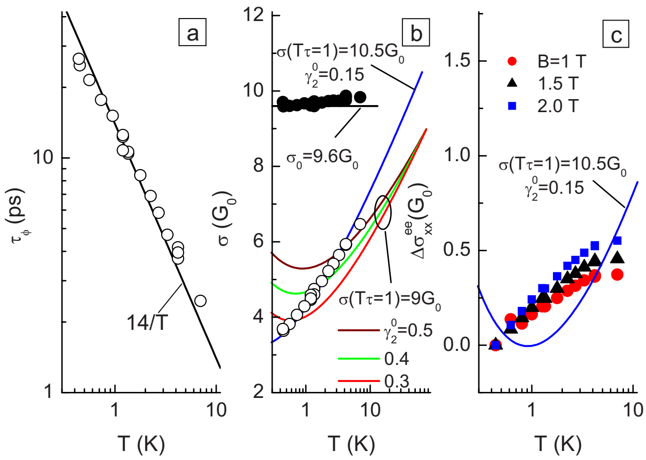

FIG. 4. (Color online) (a) The temperature dependences of the phase relaxation time, (b) σ and σ_0 found from the experiment, and (c) $\Delta\sigma_{xx}^{ee}$ taken at different magnetic field for the structure 3510 at $\sigma_0=9.6G_0$, $n=2.3\times10^{11}$ cm^{-2}. The curves in panels (b) and (c) are the solutions of the RG equations, Eqs. (2) and (3), with different initial conditions.

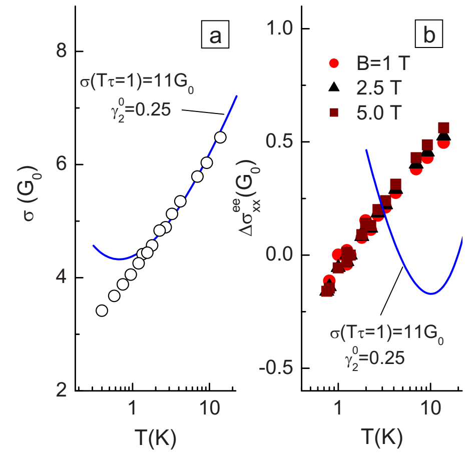

FIG. 6. (Color online) (a) The temperature dependences of the σ and (b) $\Delta\sigma_{xx}^{ee}$ taken at different magnetic field for the structure 4261 at $\sigma_0=10.2G_0$ and $n=1.05\times10^{12}$ cm^{-2}. The curves are the solutions of the RG Eqs. (2) and (3) with parameters corresponding to the best fit of the σ vs T data.

It could be assumed that the procedure of the extraction of the *e-e* interaction contribution being transparent nevertheless fails. However, the RG theory predicts that not only the *e-e* interaction contribution $\delta\sigma^{ee}$ should demonstrate the upturn with the temperature decrease at the lower Drude conductivity but the total conductivity σ as well [see Fig. 1(a)]. In this case the experiment and RG theory can be compared directly without any additional treatment of the data. Therefore, let us inspect the results for the lower Drude conductivity, $\sigma_0\simeq6.8G_0$, presented in Fig. 5. The temperature dependence of τ_ϕ found from the low-field negative magnetoresistance deviates from the $1/T$ law demonstrating tendency to saturation at low temperature [see Fig. 5(a)]. As shown in Ref. 18 such the behavior results from the fact that the dephasing length $L_\phi=\sqrt{D\tau_\phi}$ (where D is the diffusion coefficient) at low T becomes comparable with the localization length $\xi\sim l\exp(\pi k_F l/2)$, and the quantity $\tau_\phi^\star=1/(1/\tau_\phi+D/\xi^2)$, rather than τ_ϕ is experimentally obtained from the fit of the magnetoresistance. Indeed, the data in Fig. 5(a) are well described by this formula with $\tau_\phi(\mathrm{ps})=20/T$ and $\xi=15l$ that is close to $\exp(\pi k_F l/2)\simeq30l$. Since the τ_ϕ saturation is not yet observed in our temperature range, the temperature dependence of the conductivity remains close to the logarithmic one [see open symbols in Fig. 5(b)]. In contrast to that, the RG equations predict the upturn of the conductivity within our temperature range independently of the initial σ and γ_2 values, $\sigma(T\tau=1)$ and γ_2^0. As discussed above the upturn results from the strong renormalization of γ_2 that leads not only to the change in sign of $d\sigma^{ee}/dT$ but to large its value as well so that $|d\sigma^{ee}/d\ln T|>d\sigma^{\mathrm{WL}}/d\ln T$. In the actual case this results in that the calculated curve following the data at the high temperature, $T\simeq1.5-6$ K, exhibits, nevertheless, minimum at $T\simeq1.2$ K and growth at lower temperature [solid curve in Fig. 5(b)].

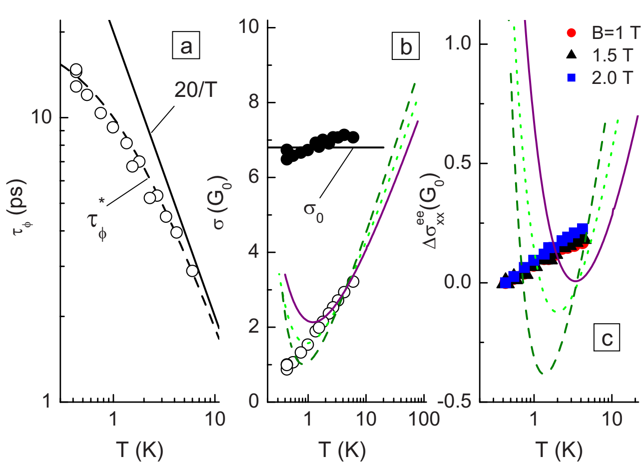

FIG. 5. (Color online) (a) The temperature dependences of the phase relaxation time, (b) σ and σ_0 found from experiment, and (c) $\Delta\sigma_{xx}^{ee}$ taken at different magnetic field for structure 3510 at $\sigma_0=6.8G_0$ and $n=2\times10^{11}$ cm^{-2}. The dashed line in (a) is the dependence $\tau_\phi^\star(T)$ with $\tau_\phi=20/T$, ps and $\xi=15l$. The lines in (b) and (c) are the solutions of the RG equations with parameters $\sigma(T\tau=1)=7.5G_0$ and $\gamma_2^0=0.12$ (solid lines), $\sigma(T\tau=1)=8G_0$, $\gamma_2^0=0.01$ (dotted lines), and $\sigma(T\tau=1)=8.5G_0$, $\gamma_2^0=-0.1$ (dashed lines).

It is clear that the one-loop approximation of the RG theory is insufficient for so low conductivity, $\sigma\sim1G_0$. It is pertinent to note here, that such decisive disagreement with the RG theory for the structure 4261 with the stronger disorder is observed at the higher conductivity (Fig. 6). As seen, the experimental T dependence of σ is close to the logarithmic one,[24] whereas the RG equations predict the upturn in the σ vs T plot already at $\sigma\simeq4.3G_0$.

Let us analyze the results in the whole. In Fig. 7 we compare the low-temperature values of the Fermi-liquid amplitude γ_2 obtained experimentally in wide range of the conductivity values, $\sigma=(2-150)G_0$,[25] with that predicted by the RG theory. The γ_2 data obtained with the help of Eq. (1) from the slope of the experimental $\Delta\sigma_{xx}^{ee}$ vs $\ln T$ dependence within the temperature range from 1.3 to 4.2 K are shown by circles. They agree well with the results obtained from the fit of the $\Delta\sigma_{xx}^{ee}$ vs T data by the RG equations, Eqs. (2) and (3)

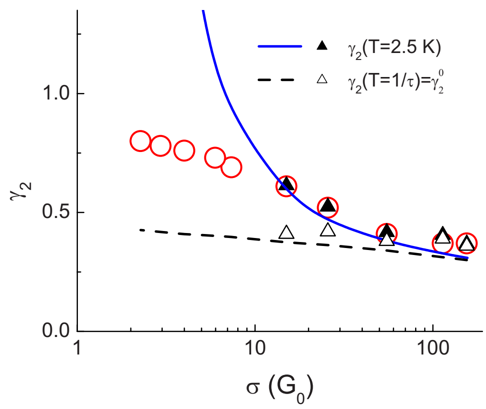

FIG. 7. (Color online) The value of γ_2 found experimentally (symbols) and calculated from the RG equations for $T=2.5$ K (solid line) as a function of the conductivity at $T=4.2$ K for the sample 3510. The dashed line is the σ dependence of γ_2^0 calculated with the help of Eq. (4). Triangles correspond to the best fit of the data by Eqs. (2) and (3). Circles are obtained with the help of Eq. (1) from the slope of the experimental $\Delta\sigma_{xx}^{ee}$ vs $\ln T$ dependence within the temperature range from 1.3 to 4.2 K.

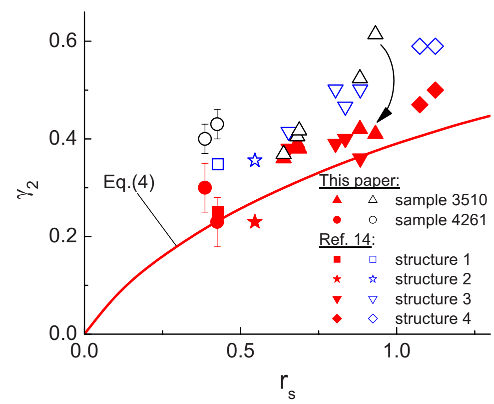

FIG. 8. (Color online) The Landau's Fermi-liquid amplitude γ_2 plotted against the gas parameter r_s for the samples 3510 and 4261, and for the structures from Ref. 14. Open symbols are obtained from the experimental value of K_{ee} at low temperature, $T=(1.3-4.2)$ K. Solid symbols are the values of γ_2^0 corresponding to the best fit of the experimental $\Delta\sigma_{xx}^{ee}$ vs T dependence by the solution of the RG equations. Arrow indicates the renormalization of γ_2 with the lowering temperature. Only the data for $\sigma>15G_0$ are presented.

(shown by solid triangles). As shown above such the fit is possible only at the relatively high conductivity, $\sigma_0 \gtrsim 18G_0$. Note that the initial values of γ_2, γ_2^0, fall closely to the curve calculated from Eq. (4). One can see that γ_2 increases monotonically when the conductivity goes down and this increase is well described by the one-loop RG equations down to $\sigma \simeq 15G_0$. At the lower conductivity, the theory predicts much steeper rise of γ_2 than that obtained from the data treatment.

As mentioned in Sec. I the behavior of the *e-e* interaction contribution with decreasing conductance was studied in Ref. 14. It has been, in particular, found that the value of K_{ee} decreases with the σ_0 decrease. We have retreated those data following the line of attack described above. The results for $\sigma>15G_0$ are presented in Fig. 8. In this figure, the open symbols are the low-temperature γ_2 value obtained from $\Delta\sigma_{xx}^{ee}$ vs T dependence. The value of γ_2^0 corresponding to the best fit of the data by the RG equations are presented by solid symbols. It is seen that while the low-temperature γ_2 points lie noticeably above the theoretical curve, Eq. (4), the γ_2^0 values fall very close to it for all the structures investigated both in Ref. 14 and in this paper.

Thus, the RG equations well describe both the temperature dependence of the conductivity at $B=0$ and the temperature dependence of the interaction contribution at the relatively high conductivity, $\sigma \gtrsim 15G_0$. The renormalization of the interaction constant γ_2 at $\sigma=15G_0$ is large enough. For instance, the amplitude γ_2 in the sample 3510 increases from 0.41 at $T=50$ K up to 0.61 at $T=2.5$ K. Such an increase corresponds to the reduction in K_{ee} by a factor of 2, from 0.46 at the higher temperature down to 0.23 at the lower one. The renormalization of γ_2 becomes stronger at further conductivity decrease. However, the experimental γ_2 vs σ plot saturates when γ_2 increases by about of two times in our conductivity range, whereas the RG theory predicts much stronger renormalization at $\sigma<10G_0$ (see Fig. 7). One of the possible reasons of the disagreement between the theory and experiment at $\sigma<15G_0$ is the restriction of the one-loop approximation and the necessity of taking into account the next terms in $1/\sigma$ expansion in RG equations.

It would seem that the understanding is achieved: the RG theory adequately describes the data while the conductivity is rather high. However, let us analyze the data in the presence of magnetic field in more detail. Equations (2) and (3) have to describe $\sigma_{xx}(T)$ if the Zeeman splitting is relatively small, $|g|\mu_B B<T$. In the case of $B>B_{tr}$, when the WL contribution is suppressed, the one unity in braces in Eq. (2) should be omitted. Because σ_{xx} strongly decreases with increasing B, $\sigma_{xx}\propto 1/B^2$, when $B>1/\mu$, the strong change in the interaction contribution due to the renormalization of γ_2 should be clearly evident also. As evident from Figs. 2(c), 3, 4(c), 5(c), and 6(b), the $\Delta\sigma_{xx}^{ee}$ vs T plots are practically independent of the magnetic field. This contradiction is more clearly illustrated by Fig. 9, where the results for structure 3510 at $\sigma_0\simeq 30.8G_0$ are presented. As seen from Fig. 9(a) σ_{xx} decreases drastically from $\simeq 25G_0$ at $B=0.5$ T down to $\simeq 2G_0$ at $B=5$ T. However, the quantity $[\delta\sigma_{xx}^{ee}(T_1)-\delta\sigma_{xx}^{ee}(4.2\text{ K})]/G_0 \ln(T_1/4.2\text{ K})$, which is K_{ee} according to Eq. (1), slightly increases against the background of the Shubnikov-de Haas (SdH) oscillations [Fig. 9(b)], instead of decreasing due to the renormalization of γ_2. It would be assumed that the decrease expected is compensated by the increase in Δ_{xx}^{ee} caused by the suppression of two from three triplet channels due to the Zeeman effect.[4,6,15,26,27] However, this effect should not be essential in our systems having $g \sim 0.5$.

To assure that the *e-e* interaction contribution does not depend on the magnetic field in spite of strong σ_{xx} decrease, we have performed analogous measurements on the structure with the very small value of g factor. It is the $Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.3}Ga_{0.7}As$ structure with the quantum well width 8 nm. The electron density in the structure is $n=8\times 10^{11}$ cm^{-2}. The value of g factor on the bottom of the conduction band for this structure is about -0.15.[28] At the

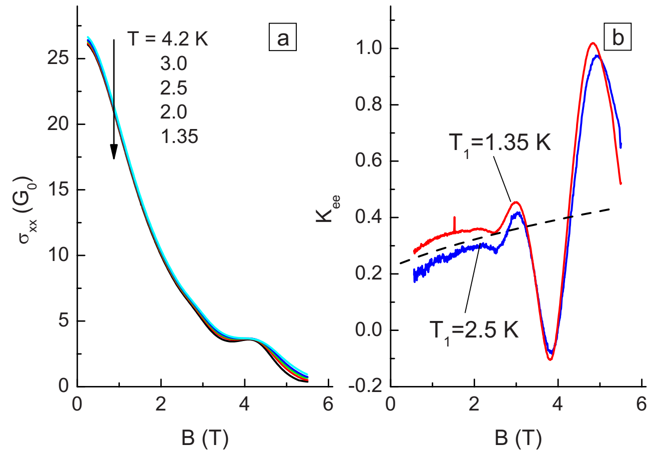

FIG. 9. (Color online) (a) The magnetic-field dependence of σ_{xx} and (b) $K_{ee}=[\sigma_{xx}^{ee}(T_1)-\sigma_{xx}^{ee}(4.2\ \text{K})]/G_0 \ln(T_1/4.2\ \text{K})$ for the structure 3510 at $\sigma_0 \simeq 30.8 G_0$ and $n=3.35\times 10^{11}\ \text{cm}^{-2}$. The dashed line indicates the monotonic run of the K_{ee} vs B curves.

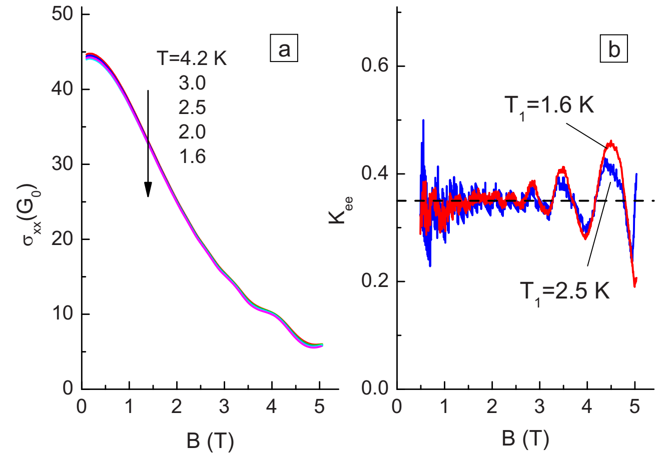

FIG. 10. (Color online) (a) The magnetic-field dependence of σ_{xx} and (b) K_{ee} for the structure $Al_{0.3}Ga_{0.7}As/GaAs/Al_{0.3}Ga_{0.7}As$ with zero g factor at $\sigma_0 \simeq 47 G_0$ and $n=8\times 10^{11}\ \text{cm}^{-2}$. The dashed line is provided as a guide for the eye, it indicates an absence of the monotonic component in K_{ee} vs B dependences.

Fermi energy, $E_F \simeq 25$ meV, its value is still less due to nonparabolicity of the conduction band, it can be estimated as $g \simeq 0 \pm 0.05$. The results are presented in Fig. 10. It is seen that even though σ_{xx} changes from $45G_0$ down to $5G_0$ in our magnetic-field range [Fig. 10(a)], there is no monotone change in the K_{ee} value [Fig. 10(b)]. Exhibiting the weak SdH oscillations it remains constant. It should be pointed out that the value of K_{ee} changes drastically when the conductivity is lowered by means of the gate voltage and it falls from $K_{ee} \simeq 0.4$ at $\sigma = 50G_0$ down to $K_{ee} \simeq 0.1$ at $\sigma = 5G_0$.

Thus, the interaction contribution to the conductivity at $B=0$ decreases when the conductivity controlled by the gate voltage goes down. This decrease is well described by the RG equations while $\sigma \gtrsim 15G_0$. On the other hand, no decrease in the interaction contribution is evident when the conductivity decreases being controlled by the external magnetic field. In this case the contribution remains constant while σ_{xx} is lowered by 1 order of magnitude. Such the behavior remains puzzling.

IV. CONCLUSION

We have experimentally studied the evolution of the diffusion part of the *e-e* interaction correction to the conductivity of 2D electron gas in $GaAs/In_{0.2}Ga_{0.8}As/GaAs$ single-quantum well within the conductivity range from $\simeq 150G_0$ down to $\simeq 1G_0$.

To separate out the interaction contribution to the conductivity we have used the fact that the *e-e* interaction contributes to the diagonal component of the conductivity tensor, σ_{xx}, only. The simultaneous analysis of the weak-localization magnetoresistance, the temperature dependence of σ_{xx} at $B \neq 0$ and σ at $B=0$ allows us to determine the conductivity dependence of the Landau's Fermi-liquid amplitude γ_2. It has been obtained that the low-temperature value of γ_2 increases with conductivity lowering. Experimental data have been compared with theoretical results obtained within the framework of the RG theory. It has been shown that the one-loop approximation adequately describes the data while the conductivity is higher than $\simeq 15G_0$. At lower conductivity, drastic disagreement between theory and experiment is evident, suggesting the next-loop approximations in the RG theory are needed. Finally, it remains unclear why the renormalization of the Fermi-liquid amplitude takes experimentally place when the conductivity decreases with the gate voltage but does not when it decreases in the external magnetic field. More work is required to resolve this issue.

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[24] By and large the structures 4261 and 3510 behave themselves analogously. Exception is that the slope of the σ vs ln T dependence is somewhat different from that predicted theoretically, $1+K_{ee}$. This issue will be discussed elsewhere.
[25] We have supposed that the change in the slope of σ vs ln T and σ_{xx} vs ln T dependences results from the renormalization of the interaction constant only. It is not straightforward method for the determination of the interaction constant. More direct method is based on the suppression of the triplet channel in the magnetic field due to the Zeeman effect. This method is applied for the systems with relatively large magnitude of g factor. It was used for the 2D hole system (Ref. 15) where $g \simeq 1.5-2$. However, the holes are the carriers of four-fold degenerated band with the total moment 3/2. Therefore it is not quite correct to compare the experimental data with the predictions of the RG theory developed for 2D electron systems with the simple, single-valley energy spectrum.
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